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Abstracts of Papers to Appear

MONOTONICITY PRESERVING WEIGHTED ESSENTIALLY NONOSCILLATORY SCHEMES WITH INCREASINGLY HIGH ORDER OF ACCURACY. Dinshaw S. Balsara* and Chi-Wang Shu.† **N.C.S.A., University of Illinois at Urbana-Champaign 605 E. Springfield Avenue, Champaign, Illinois 61820; and †Division of Applied Mathematics, Brown University Providence, Rhode Island 02912.* E-mail: dbalsara@ncsa.uiuc.edu, shu@cfm.brown.edu.

In this paper we design a class of numerical schemes that are higher order extensions of the weighted essentially nonoscillatory (WENO) schemes of Jiang and Shu [23] and Liu, Osher, and Chan [27]. Used by themselves, the schemes may not be always monotonicity preserving but coupled with the monotonicity preserving bounds of Suresh and Huynh [45] they perform very well. The resulting monotonicity preserving weighted essentially nonoscillatory (MPWENO) schemes have high phase accuracy and high order of accuracy. The higher order members of this family are almost spectrally accurate for smooth problems. They, nevertheless, have robust shock capturing ability. The schemes are stable under normal CFL numbers. They are also efficient and do not have a computational complexity that is substantially greater than that of the lower order members of this same family of schemes. The higher accuracy that these schemes offer coupled with their relatively low computational complexity makes them viable competitors to lower order schemes such as the older total variation diminishing (TVD) schemes, for problems containing both discontinuities and rich smooth region structure. We describe the MPWENO schemes here as well as show their ability to reach their designed accuracies for smooth flow. We also examine the role of steepening algorithms such as the artificial compression method (ACM) in the design of very high order schemes. Several test problems in one and two dimensions are presented. For multidimensional problems where the flow is not aligned with any of the grid directions it is shown that the present schemes have a substantial advantage over lower order schemes. It is argued that the methods designed here have great utility for direct numerical simulations (DNS) and large eddy simulations (LES) of compressible turbulence. The methodology developed here is applicable to other hyperbolic systems and that is demonstrated by showing that the MPWENO schemes also work very well on magnetohydrodynamical (MHD) test problems.

NEW HIGH-RESOLUTION CENTRAL SCHEMES FOR NONLINEAR CONSERVATION LAWS AND CONVECTION-DIFFUSION EQUATIONS. Alexander Kurganov* and Eitan Tadmor.† **Department of Mathematics, University of Michigan, Ann Arbor, Michigan 48109; and †Department of Mathematics, University of California—Los Angeles, Los Angeles, California 90095.* E-mail: kurganov@math.lsa.umich.edu, tadmor@math.ucla.edu.

Central schemes may serve as universal finite-difference methods for solving nonlinear convection-diffusion equations in the sense that they are not tied to the specific eigen-structure of the problem, and hence can be implemented in a straightforward manner as black-box solvers for general conservation laws and related equations governing the spontaneous evolution of large gradient phenomena. The first-order Lax-Friedrichs scheme [29] is the forerunner for such central schemes. The central Nessyahu-Tadmor (NT) scheme [38] offers higher resolution while retaining the simplicity of Riemann-solver-free approach. The numerical viscosity present in these central schemes is of order $\mathcal{O}((\Delta x)^{2r}/\Delta t)$. In the convective regime where $\Delta t \sim \Delta x$, the improved resolution of NT scheme and its generalizations is achieved by lowering the amount of numerical viscosity with increasing r. At the same time, this family of central schemes suffers from excessive numerical viscosity when a sufficiently small time step is enforced, e.g., due to the presence of degenerate diffusion terms. In this paper we introduce a new family of central schemes which retain the simplicity of being independent of the eigen-structure of the problem, yet they enjoy a much smaller numerical viscosity (of the corresponding order $\mathcal{O}(\Delta x)^{2r-1}$)). In particular, our new central schemes maintain their high-resolution independent of $\mathcal{O}(1/\Delta t)$, and letting $\Delta t \downarrow 0$, they admit a particularly simple semi-discrete formulation. The main idea behind the construction of these central schemes

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is the use of more precise information of the *local propagation speeds*. Beyond these CFL related speeds, no characteristic information is required. As a second ingredient in their construction, these central schemes realize the (nonsmooth part of the) approximate solution in terms of its cell averages integrated over the Riemann fans of *varying size*. The semi-discrete central scheme is then extended to multidimensional problems, with or without degenerate diffusive terms. Fully discrete versions are obtained with Runge-Kutta solvers. We prove that scalar version of our high-resolution central scheme is nonoscillatory in the sense of satisfying the TVD property in the one-dimensional case and the maximum principle in two-space dimensions. We conclude with a series of numerical examples, considering convex and nonconvex problems with and without degenerate diffusion, and scalar and systems of equations in one- and two-space dimensions. The time evolution is carried out by the third- and fourth-order explicit embedded integration Runge-Kutta methods recently proposed by Medovikov [37]. These numerical studies demonstrate the remarkable resolution of our new family of central scheme.

A DIRECT SPECTRAL COLLOCATION POISSON SOLVER IN POLAR AND CYLINDRICAL COORDINATES. Heli Chen,* Yuhong Su,* and Bernie D. Shizgal.*,† **Institute of Applied Methematics, and †Department of Chemistry, University of British Columbia, Vancouver, British Columbia, Canada.* E-mail: hchen@math.ubc.ca, George@seagatesoftware.com, shizgal@theory.chem.ubc.ca.

In this paper, we present a direct spectral collocation method for the solution of the Poisson equation in polar and cylindrical coordinates. The solver is applied to the Poisson equations for several different domains including a part of a disk, an annulus, a unit disk, and a cylinder. Unlike other Poisson solvers for geometries such as unit disk and cylinder, no pole condition is invovled for the present solver. The method is easy to implement, fast, and gives spectral accuracy. We also use the weighted interpolation technique and nonclassical collocation points to improve the convergence.

EXPANSION FUNCTIONS FOR TWO-DIMENSIONAL INCOMPRESSIBLE FLUID FLOW IN ARBITRARY DOMAINS. J. P. Goedbloed. *FOM-Institute for Plasma Physics 'Rijnhuizen', Association Euratom-FOM, 3439MN Nieuwegein, The Netherlands.* E-mail: goedbloed@rijnh.nl.

Expansion functions are presented for two-dimensional incompressible fluid flow in arbitrary domains that optimally conserve the 2D structure of vortex dynamics. This is obtained by conformal mapping of the domain onto a circle and by constructing orthogonal radial polynomials and angular harmonics on the new domain such that the kinetic energy is diagonal and the separate components satisfy all of the required physical boundary conditions.

OVERLAPPING SCHWARZ METHODS FOR UNSTRUCTURED SPECTRAL ELEMENTS. Luca F. Pavarino* and Timothy Warburton.† **Department of Mathematics, Università di Milano Via Saldini 50, 20133 Milano, Italy; †Oxford University Computing Laboratory, Wolfson Building, Parks Road, Oxford, OX1 3QD, United Kingdom.* E-mail: pavarino@ares.mat.unimi.it, timw@comlab.ox.ac.uk.

A parallel and scalable domain decomposition method for unstructured and hybrid spectral element discretizations of elliptic problems is introduced and studied. The spectral elements are affine images of the reference triangle or square in two dimensions and of the reference tetrahedron, pyramid, prism, or cube in three dimensions. The method is based on overlapping Schwarz techniques applied to the Schur complement of the discrete system and is implemented as a preconditioner for a Krylov space method. Numerical results in two and three dimensions show that the iteration counts of our method are bounded by a constant independent of the spectral degree and the number of subdomains. The resulting elliptic solver can be used in Navier–Stokes simulations using the spectral element code NekTar.